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Nutation Damper Instability on Spin-Stabilized Spacecraft

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The dynamic equations describing the motion of a single-degree-of-freedom nutation damper mounted on a dual-spin spacecraft are examined. The motion of the damper mass is found to be parametrically excited by the transverse angular rates, so that instabilities of this motion are possible. In this situation, instability could imply that the damper mass moves to an extreme position and remains there; hence, it would not perform its intended function of dissipating energy in order to damp out nutation. It is found that this unstable behavior is quite possible for a damper mounted on a single-spin spacecraft or on the spinning portion of a dual-spin spacecraft where the other member is essentially despun. On the other hand, unstable behavior of a damper mounted on the despun member of a dual-spin configuration is found to be extremely unlikely.

Nomenclature

A, C = moments of inertia, slug ft^2 (ft-lb-sec^2)
D = angular momentum ratio
H = angular momentum, ft-lb-sec
M = moment, ft-lb
P = amplitude of angular velocities, rad/sec
a = acceleration, ft/sec^2
b = length, ft
c = damping constant, lb-sec/ft
d,e = parameters
f_n = natural frequency, rad/sec
k = spring constant, lb/ft
m = mass, slugs (lb-sec^2/ft)
v = velocity, fps
δ,ϵ = parameters
ζ = damping ratio
θ = wobble angle, rad
μ = characteristic exponent
τ = nondimensional time
Ω = precessional frequency, rad/sec
ω = angular velocity, rad/sec

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Introduction

MANY passive nutation dampers (passive implying that the damper is driven by the coning motion of the spacecraft) have been proposed for eliminating undesirable nutational motions of spin-stabilized spacecraft.[1–3] The analyses for most of these designs involve many approximations and linearizations, since they are intended to provide design guidelines for the dampers. However, some analysts have examined the complete set of equations for certain dampers and have uncovered nonlinear and even unstable forms of motion.[4,5] An important aspect of any damper application, then, is the study of any possible nonlinear behavior or instability in its motion. In this paper, a very simple, one-degree-of-freedom, mass-spring-dashpot damper is analyzed. This type of damper has been studied previously (by a linearized analysis) for a single-spin system,[6] but in the present case, the analysis is expanded to include dual-spin systems, i.e., spacecraft consisting of two major bodies which rotate relative to one another about a common spin axis. A form of instability of the motion of the damper mass is pointed out and discussed, and criteria for the onset of this unstable motion are established.

Complete Equations of Motion

The configuration analyzed is shown in Fig. 1. For simplification, it is assumed that both bodies are inertially symmetric about their common spin axes, the z and ζ axes. The spin moments of inertia, C_1 and C_2, of the bodies and the transverse moments, A_1 and A_2, are referenced to the composite center of mass of the two main bodies. The complete system center-of-mass actually shifts relative to the spin axis due to the motion of the damper mass, but Fang[7] has shown that this moving mass can be rigorously accounted for by retaining as the origin of coordinates the center-of-mass of the main bodies and by introducing a "reduced mass" for the moving damper mass equal to

$$m = m_d(1 - m_d/m_s) \tag{1}$$

where m_d is the actual damper mass and m_s is the total mass of the spacecraft. The equations of motion will be written relative to the x,y,z triad fixed in body 2, the body which contains the damper.

With the definition $A_1 + A_2 = A$, the angular momentum components are

$$\left.\begin{aligned} H_x &= A\omega_x - mbv_y \\ H_y &= A\omega_y + mbv_x - mxv_z \\ H_z &= C_1\omega_\zeta + C_2\omega_z + mxv_y \end{aligned}\right\} \tag{2}$$

where v_x, v_y, and v_z are the linear velocity components of the damper mass relative to the composite center of mass and are given by

$$\left.\begin{aligned} v_x &= \dot{x} + b\omega_y \\ v_y &= x\omega_z - b\omega_x \\ v &= -x\omega_y \end{aligned}\right\} \tag{3}$$

Substituting into Euler's equations

$$\dot{H}_i + \omega_j H_k - \omega_k H_j = 0$$

where i, j, and k represent cyclic permutations of x, y, and z, yields

$$A\dot{\omega}_x + [C_1\omega_\zeta + (C_2 - A)\omega_z]\omega_y = mb[2\dot{x}\omega_z + b(\omega_y\omega_z - \dot{\omega}_x) + x(\omega_x\omega_y + \dot{\omega}_z)] \tag{4}$$

$$A\dot{\omega}_y - [C_1\omega_\zeta + (C_2 - A)\omega_z]\omega_x = -m[b\ddot{x} + 2x\dot{x}\omega_y + b^2(\omega_x\omega_z + \dot{\omega}_y) - x^2(\omega_x\omega_z - \dot{\omega}_y) + bx(\omega_x^2 - \omega_z^2)] \tag{5}$$

$$C_1\dot{\omega}_\zeta + C_2\dot{\omega}_z = -mx[2\dot{x}\omega_z + b(\omega_y\omega_z - \dot{\omega}_x) + x(\omega_x\omega_y + \dot{\omega}_z)] \tag{6}$$

The acceleration of the damper mass in the x direction is

$$a_x = \ddot{x} - x(\omega_y^2 + \omega_z^2) + b(\omega_x\omega_z + \dot{\omega}_y)$$

and from Newton's law

$$a_x + (c/m)\dot{x} + (k/m)x = 0$$

so that

$$\ddot{x} + (c/m)\dot{x} + (k/m - \omega_z^2 - \omega_y^2)x = -b(\dot{\omega}_y + \omega_x\omega_z) \tag{7}$$

Equations (4–7) form a set of four equations for the five variables ω_x, ω_y, ω_z, ω_ζ, and x; another condition or equation is required. One could, for example, introduce an equation describing the transfer function of a motor drive and control system connecting the two bodies. Alternatively, one of the bodies could be assumed to be perfectly despun. A somewhat different assumption will be made in order to obtain an approximate solution to the equations, as described later.

Consider the coefficient of x in Eq. (7). The apparent spring constant of the damper is diminished by the centrifugal force due to spin about the z axis (the ω_z^2 term) and also by the contribution of the cross spin centrifugal force (the ω_y^2 term). Although the latter term may be small compared to the first terms, it cannot be neglected, since this periodic force can cause parametric excitation of the damper mass motion. In fact, the homogeneous part of Eq. (7) bears a strong resemblance to Hill's equation, which has been extensively studied.[8] Forms of this type of equation can lead to instability of the motion, i.e., the variable x could grow without bound. Similar instabilities have been uncovered previously. Kane[9] and Thomson and Fung[10] have analyzed the dynamics of spinning bodies with auxiliary translating particles which involve parametric excitation. However, in these references, the auxiliary particle parametrically excites the main body through Coriolis forces; conversely, the present problem concerns the main body exciting an auxiliary particle by centrifugal forces. Since Eqs. (4–7) cannot be solved analytically, an approximate solution will be obtained in order to derive criteria for the combinations of parameters which could lead to instability. Digital computer solutions of special cases of the full set of equations, which were used to verify the instability criteria, will be discussed subsequently.

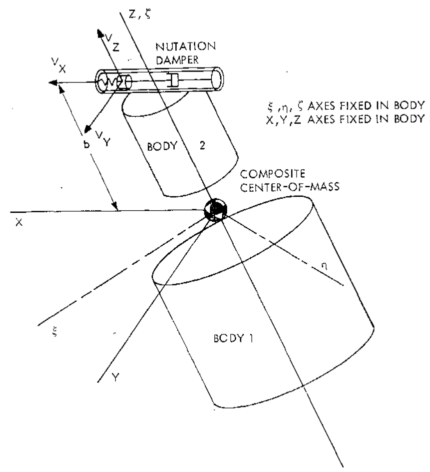

Fig. 1 Spacecraft dynamics parameters.

Approximate Solution

The effects of the damper moments are usually very small during one cycle of rotation, so that the homogeneous solutions of Eqs. (4–6) will serve to describe the motion of the basic system quite adequately for several cycles. (This is the assumption used in most approximate treatments.) Hence, the present problem is to solve the set of equations

$$A\dot{\omega}_x + [C_1\omega_\zeta + (C_2 - A)\omega_z]\omega_y = 0 \tag{8}$$

$$A\dot{\omega}_y - [C_1\omega_\zeta + (C_2 - A)\omega_z]\omega_x = 0 \tag{9}$$

$$C_1\dot{\omega}_\zeta + C_2\dot{\omega}_z = 0 \tag{10}$$

The necessary additional condition chosen herein is that the kinetic energy of the system is essentially constant for several cycles of spin, which implies that any motor provided between two bodies only serves to compensate for frictional losses in the mutual bearings. It can be shown by consideration of the energy integral of the equations that this condition implies that ω_ζ and ω_z are both constant for several cycles. (Hence, this assumption includes the perfectly despun situation as a special case.) Then, defining the constant parameter

$$\Omega = [C_1\omega_\zeta + (C_2 - A)\omega_z]/A \tag{11}$$

the solutions to Eqs. (8) and (9) are evidently

$$\omega_x = P\sin\Omega t \tag{12}$$

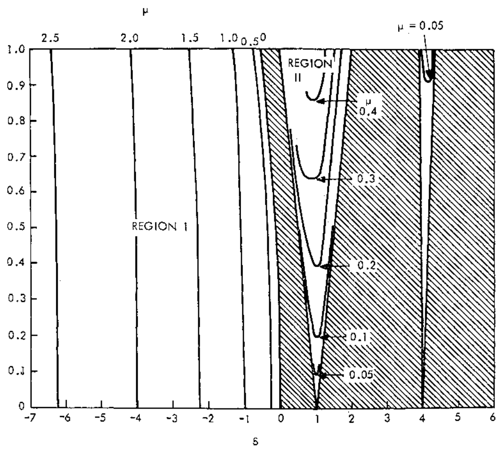

Fig. 2 Mathieu equation stability diagram (the cross-hatched areas are regions of stability).

$$\omega_y = -P \cos\Omega t \tag{13}$$

The components of angular momentum of this system are[11]

$$H \sin\theta = A(\omega_x^2 + \omega_y^2)^{1/2} \tag{14}$$

$$H \cos\theta = C_1\omega_\zeta + C_2\omega_z \tag{15}$$

where θ is the "wobble angle," i.e., the angle between the spin axis of the spacecraft and the angular momentum vector. Hence

$$P = (H/A) \sin\theta \tag{16}$$

$$\Omega = (H/A) \cos\theta - \omega_z \tag{17}$$

It can be shown[11] that the quantity Ω must be positive to insure stability of rotation. If Ω were negative, then the damper would tend to tumble the vehicle rather than to restore pure spin.

Turning now to the motion of the damper, the homogeneous part of Eq. (7) is

$$\ddot{x} + (c/m)\dot{x} + (k/m - \omega_z^2 - \omega_y^2)x = 0 \tag{18}$$

Defining

$$f_n^2 = k/m - \omega_z^2 \tag{19}$$

and using Eq. (13) (and double-angle formulas), then

$$\ddot{x} + (c/m)\dot{x} + [(f_n^2 - P^2/2) - (P^2/2) \cos 2\Omega t]x = 0 \tag{20}$$

Introducing nondimensional time

$$\tau = \Omega t$$

and the damping ratio

$$\zeta = c/2mf_n \tag{21}$$

then we have

$$\frac{d^2x}{d\tau^2} + 2\zeta\left(\frac{f_n}{\Omega}\right)\frac{dx}{d\tau} + \left[\frac{f_n^2 - P^2/2}{\Omega^2} - \left(\frac{P^2}{2\Omega^2}\right)\cos 2\tau\right]x = 0 \tag{22}$$

In order to put the equation into a standard form, it will be necessary to eliminate the first derivative term. This can be done by introducing the transformation[12]

$$x = \alpha \exp[-\zeta(f_n/\Omega)\tau] \tag{23}$$

which, with $\dot{\alpha} \equiv d\alpha/d\tau$, etc., yields

$$\ddot{\alpha} + \left[(1 - \zeta^2)\left(\frac{f_n}{\Omega}\right)^2 - \frac{P^2}{2\Omega^2} - \left(\frac{P^2}{2\Omega^2}\right)\cos 2\tau\right]\alpha = 0 \tag{24}$$

Finally, if we define

$$2\epsilon = P^2/2\Omega^2 \tag{25}$$

and

$$\delta = (1 - \zeta^2)(f_n/\Omega)^2 - 2\epsilon \tag{26}$$

then

$$\ddot{\alpha} + (\delta - 2\epsilon \cos 2\tau)\alpha = 0 \tag{27}$$

Equation (27) can now be recognized as a standard form of Hill's equation known as the Mathieu equation. A stability chart for this equation has been generated for a large range of values of δ and ϵ.[8] The relevant area of the stability chart of interest here is shown on Fig. 2. If the parameters in a particular problem fall into a region of instability on this graph, then α grows without bound; however, the solution for x may still be stable. This is true since the variable α is multiplied by a decreasing exponential in Eq. (23). Unstable solutions for α are always of the form

$$\alpha = F(\tau) \exp(\mu\tau) \tag{28}$$

where $F(\tau)$ is a periodic function.[13] Hence, the solution for the initial variable, x, will be of the form

$$x = F(\tau) \exp[(\mu - \zeta f_n/\Omega)\tau] \tag{29}$$

so that the criterion for stability becomes

$$(\mu - \zeta f_n/\Omega) \leq 0 \tag{30}$$

Lines of constant μ, the "characteristic exponent" (obtained from Ref. 14), are shown on Fig. 2. For any given design, the parameters δ and ϵ can be found using relations (25) and (26); then Fig. 2 can be consulted to determine if there is any instability and, if so, the corresponding value of μ. Finally, Eq. (30) can be used to determine the stability of the actual system.

For all systems investigated, any possible instability would occur in one of the two regions labeled on Fig. 2. In region I, the cross-spin centrifugal force term, i.e., the ω_y^2 term in Eq. (7), becomes large enough to make the apparent spring constant of the damper negative during part of the motion. This negative spring constant leads to a divergent motion of the damper mass, a monotonically increasing deflection with a small, superimposed oscillation, as shown in Fig. 3. In region II of Fig. 2, the apparent spring constant is always positive, but here the cross-spin term increases the damper mass motion during each cycle, so that x will typically be an oscillation with an exponentially increasing amplitude (flutter). This characteristic motion is suggested by the

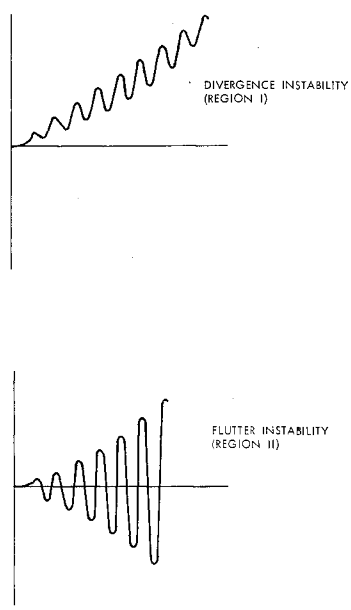

Fig. 3 Types of instability.

lower sketch in Fig. 3. For an actual physical damper, where motion of the damper mass would be restricted by the walls of a container or by built-in stops, this implies that for instabilities in region I, the damper mass would move to an extreme position and remain there so that it would not perform its intended function of dissipating energy.

In region II, on the other hand, the damper mass would continue to oscillate even if it bounced off the walls of its container. With suitable cushions at these walls, this type of motion could conceivably be desirable, since it would tend to dissipate a great deal of energy. However, Fig. 2 and Eq. (30) indicate that even for low values of damping, large nutation angles would be necessary to cause instability (to bring the design point above the critical μ curve).

Application of Criteria to Sample Cases

It will prove instructive to examine some specific applications of the foregoing criteria to determine whether or not some of these instability modes are actually possible in typical designs.

With the aid of Eqs. (15–17), the important parameters ϵ and δ, Eqs. (25) and (26), can be put into the form

$$\epsilon = (D \tan^2\theta)/4 \tag{31}$$

and

$$\delta = (1 - \zeta^2)[f_n/(H/A)]^2[4\epsilon + D] - 2\epsilon \tag{32}$$

where

$$D = 1/[1 - A\omega_z/(C_1\omega_\zeta + C_2\omega_z)]^2 \tag{33}$$

There are three important cases which can now be discussed. They are as follows.

Case 1 Dual-spin spacecraft with one member despun; damper mounted on despun portion. This case is exemplified by the OSO satellite.[15]

Case 2 Dual-spin spacecraft with one member despun; damper mounted on spinning portion. The ITS-III communications satellite belongs to this category, since the nutation damper is on the basic (spinning) spacecraft and not on the mechanically despun antenna. (Note, however, that the motion of the particular damper now used on this spacecraft is not described by the damper equations derived herein.[16])

Case 3 Single, rigid body spacecraft.

The damper has been assumed always mounted on body 2, so that case 1 is characterized by $\omega_z = 0$, case 2 by $\omega_\zeta = 0$, and case 3 by $\omega_\zeta = \omega_z$.

For case 1, with $\omega_z = 0$, D becomes equal to 1. Then, Eq. (31) indicates that even for moderate nutation angles, the quantity ϵ will be restricted to very small numbers. Since for instabilities to occur in region II large values of ϵ and small damping are required, it seems unlikely that this type of instability could occur. Thus, only instabilities in region I are of concern here.

In region I, the near linearity of the μ curves allows a simple criterion to be derived for instability (for small ϵ). Incidentally, this simple criterion is not restricted to despun systems; it holds in general. It can be seen from Fig. 2 that in region I

$$\delta \approx -\mu^2 \tag{34}$$

Using Eqs. (25, 26, and 30), the stability criterion becomes

$$P^2 - 2f_n^2 \geq 0$$

or, with Eq. (16)

$$(H/A) \sin\theta \leq (2)^{1/2}f_n \tag{35}$$

It is of interest that this criterion is independent of the damping ratio.

For efficient operation, the damper could be "tuned" to the nutation rate Ω at zero nutation angle

$$f_n \approx H/A \tag{36}$$

Hence, the stability criterion becomes

$$\sin\theta \leq (2)^{1/2} \tag{37}$$

which is, of course, always true. Suppose that the system is designed for despin, but that the despun portion loses its reference signal and begins to speed up due to friction. In this situation, the apparent natural frequency of the damper is given by Eq. (19), where, if the system has been tuned to despun conditions

$$k/m = (H/A)^2$$

Thus, Eq. (35) can be written

$$(H/A) \sin\theta \leq (2)^{1/2}[(H/A)^2 - \omega_z^2]^{1/2} \tag{38}$$

The maximum allowable value of ω_z is that which causes Ω to go to zero, since larger ω_z will result in the relative nutation rate, Ω, going negative, which leads to over-all attitude drift instability of the whole system. From Eq. (17)

$$(\omega_z)_{\max} = (H/A) \cos\theta$$

so that Eq. (38) becomes

$$1 \leq (2)^{1/2} \tag{39}$$

which is always true.

Thus, this type of instability is not a problem for a dual-spin system in which one member is despun and where the despun-body-mounted nutation damper is approximately tuned to the nutation frequency of the system. Some caution may still be required if the damper cannot be turned to the nutation frequency for physical reasons, or if the system is operating in some off-design mode (e.g., if the actual spin rate turns out to be quite different from the design spin rate).

Turning to cases 2 and 3, with $\omega_\zeta = 0$, the parameter D for case 2 is given by

$$D = [C_2/(C_2 - A)]^2 \tag{40}$$

while for case 3, with $\omega_1 = \omega_z$, and $C_1 + C_2 = C$

$$D = [C/(C - A)]^2 \tag{41}$$

Since Eqs. (42) and (43) are the same in form, these two cases can be discussed simultaneously. For convenience, case 3 will be discussed, but the conclusions will also hold for case 2 with the quantity C replaced by C_2.

For vehicles in which C is only slightly greater than A,† D can be quite large; hence, the quantity ϵ can also be a large number, even for small nutation angles. This means that instabilities in both regions I and II are possible.

For example, in region I, the stability criterion, Eq. (35), can be written

$$(C/A)\omega_\zeta \tan\theta \leq (2)^{1/2}f_n \tag{42}$$

If we again tune the damper such that

$$f_n = \Omega(\theta = 0) = [(C/A) - 1]\omega_\zeta/\cos\theta$$

the criterion reduces to

$$\sin\theta \leq (2)^{1/2}[(C - A)/C] \tag{43}$$

Now, if C is not much greater than A, instability could indeed occur, which is in contrast to case 1, in which instability was almost impossible.

† It will be remembered that C must be larger than A for over-all system stability since now, from Eq. (11), $\Omega = [(C - A)/A]\omega_\zeta$.

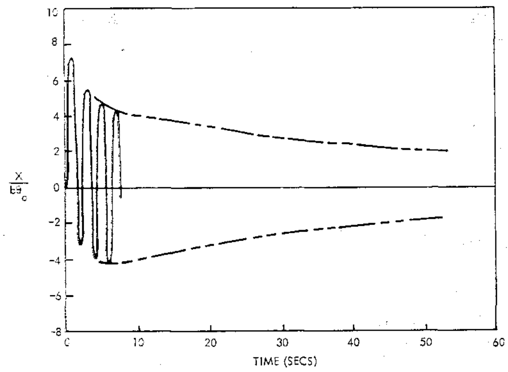

Fig. 4 Time history of damper mass motion, $\theta_0 = 10°$.

Exact Solution

To determine just what effect the "instability" which has been uncovered has on a system, the complete set of equations of motion for case 3, i.e., Eqs. (4–7), with $\omega_\zeta = \omega_z$, were programed for solution on a digital computer. The particular parameter values chosen for the runs were selected from among the cases discussed by Wadleigh et al.[6] and by Slachmuylders[17] in his comment on the Wadleigh et al. paper. These two references were concerned with linearized solutions to this set, so that the computer solution also served as a check on the accuracy of the linearized equations. The parameter values are given in Table 1.

The parameters for case 3 fall into region I of the stability diagram (Fig. 2). Substituting the values listed in Table 1 into Eq. (42) leads to a critical nutation angle of 18.4° as the boundary of instability, i.e., the motion of the system should change radically if it is started with a nutation angle greater than 18.4°.

Computer runs were made using initial nutation angles of 2°, 10°, and 20°. The resulting damper motion for a 10° nutation angle is shown in Fig. 4(the motions for an angle of 2° are essentially identical except for minor differences in the starting transient). Figure 5 shows the damper mass motion when the starting angle is 20°; the divergent behavior is strikingly evident. However, after starting out with a strongly divergent motion, the damper mass settles down to an off-center oscillation.

If the damper design had been based on the linearized analysis, this behavior would have been unsuspected, and the case designed to hold the damper mass and damping fluid would probably have been designed to be slightly longer than the maximum expected excursion. In that event, the divergent motion of the damper mass for a 20° angle would actually be container-limited. A typical result for this situation is depicted in Fig. 6 (the container was simulated on the computer by stepwise increases in the damping and spring constants).

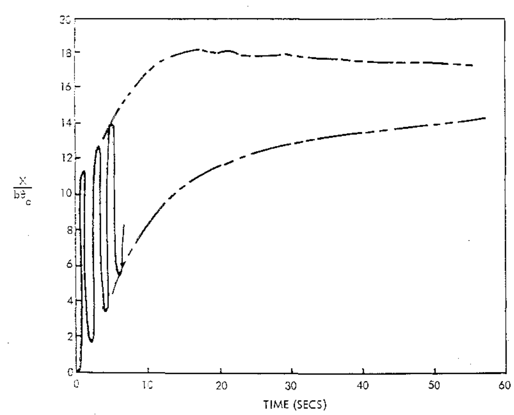

Fig. 5 Time history of damper mass motion, $\theta_0 = 20°$.

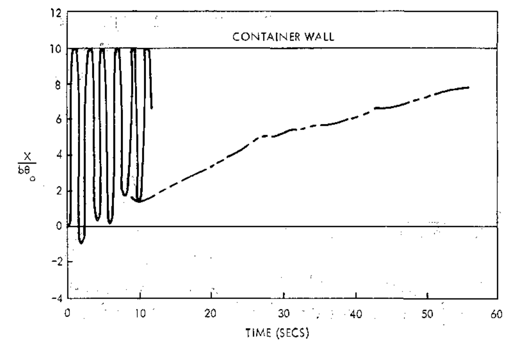

Fig. 6 Time history of damper mass motion, $\theta_0 = 20°$, container-limited.

The resulting effects of these damper motions on the system are shown in Fig. 7. Here, the rates of decay of one of the transverse angular rates are plotted for each of the initial nutation angles. (These curves are the envelopes of the rates, which are oscillatory.) The "time constant," i.e., the time for the transverse rate to decrease to $1/e$ of its initial value, for the 20° angle, container-limited case is 12% higher than for the 2° and 10° angle cases, and almost 20% higher than the 20°, non-container-limited case. Of course, different assumptions about the characteristics of the container wall could alter the results even more drastically.

Both of the cases started at a 20° nutation angle will converge to unsymmetric configurations in which the damper mass will end up in an off-center position, the spring force being balanced by centrifugal force. Thus, the main body will not end up in pure spin about its own principal axis, but instead will be spinning about a displaced axis. What has happened in this particular case is that there exists a new equilibrium state of the system, described by

$$\omega_y = \dot{\omega}_y = \dot{\omega}_x = \dot{\omega}_z = \dot{x} = \ddot{x} = 0 \tag{44}$$

$$\omega_x^2 = \left(1 - \frac{\omega_z^2}{k/m}\right)\left[\frac{k}{m} - \left(\frac{k}{m} - \omega_z^2\right)\frac{(C-A)}{mb^2}\right] \tag{45}$$

$$x/b = -\omega_x\omega_z/(k/m - \omega_z^2) \tag{46}$$

as can be verified by direct substitution into the exact equations of motion. A similar equilibrium state has been found for this type of system previously,[18] but in that situation, the basic motion of pure spin about the symmetry axis was found to be unstable. Here, both the pure spin and this new, displaced orientation can exist; the damper instability has driven the system into the latter state. The ultimate question of stability of this new state of motion involves consideration of other damping mechanisms (such as hysteresis or fuel sloshing in the main satellite bodies) and is beyond the scope of the present discussion.

It should be mentioned that the results for the 2° and 10° cases verified the prediction of Slachmuylders[17]; the transverse rate decreased to $\frac{1}{10}$ of its initial value in 120 sec just as shown on Fig. 1b of Ref. 14. This agreement indicates that linearizing the equations of motion is valid for damper design

Table 1 Parameter values for computer solution of case 3 equations of motion

Parameter	Value
C/A	1.36
mb^2/A	6.46×10^{-4}
ω_z	8.79 rad/sec
$(k/m)^{1/2a}$	9.22 rad/sec
λ^b	0.5

[a] This damper is not "tuned"; its natural frequency while spinning is 90% of the nutation frequency.

[b] This parameter λ is the damping ratio based on the nonspinning natural frequency of the damper, $(k/m)^{1/2}$.

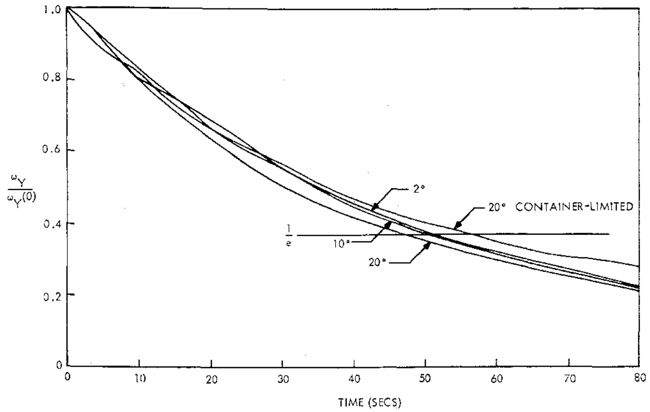

Fig. 7 Time history of transverse angular rates.

purposes, as long as the initial nutation angle is less than the critical angle for instability.

Discussion and Conclusions

The form of the equation governing the motion of this particular nutation damper mounted on a dual-spin spacecraft does admit of possible unstable motions. However, for a spacecraft with a despun member and a tuned damper mounted on the despun member, it has been found that instabilities can occur only for off-design conditions very different from typical nominal design conditions.

Included in this category (a despun-member-mounted damper) are spacecraft whose moments of inertia are such that they can be unstable in attitude if the damper is not operating. Hence, tumbling could occur if the nutation damper were indeed subject to divergent instability, so that the improbability of unstable damper behavior is a fortuitous result.

On the other hand, if the damper is mounted on the spinning portion of a dual-spin spacecraft or for the special case of a single rigid body spacecraft, it is quite possible for instabilities of the damper's motion to occur. These instabilities could appear if the nutation angle became large enough to violate the stability criteria, which could occur during attitude reorientation maneuvers, for example. Since such satellites must be designed to be stable against tumbling, then the damper instability will not tumble the vehicle, but may drive it into a new, undesirable equilibrium state. Since many other specific types of damper have motions described by differential equations with periodic coefficients (before linearization), the possibility of parametric excitation of the motion of these dampers should be checked carefully in all spin-stabilized satellite designs.

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